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ECpE Department

EE653 Power distribution system
modeling, optimization and
simulation

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Modeling Shunt Components – Loads and Caps

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Load Models

The loads on a distribution system are typically specified by the complex power consumed. This demand can be specified as kVA and power factor, kW and power factor, or kW and kvar. The voltage specified will always be the voltage at the low-voltage bus of the distribution substation. This creates a problem since the current requirement of the loads can not be determined without knowing the voltage. For this reason, modified ladder iterative technique must be employed.

Loads on a distribution feeder can be modeled as wye connected or delta connected. The loads can be three phase, two phase, or single phase with any degree of unbalanced. The *ZIP* models are

- Constant impedance (Z)
- Constant current (I)
- Constant real and reactive power (constant P)
- Any combination of the above

Load Models

- The load models developed are to be used in the iterative process of a power-flow program where the load voltages are initially assumed.
- One of the results of the power-flow analysis is to replace the assumed voltages with the actual operating load voltages.
- All models are initially defined by a complex power per phase and an *assumed* line-to-neutral voltage (wye load) or an assumed line-to-line voltage (delta load). The units of the complex power can be in volt-amperes and volts or per-unit volt-amperes and per-unit voltages.
- For all loads, the line currents entering the load are required in order to perform the power-flow analysis.
- *Actually, for all shunt components, including static loads, caps and induction machines, what we need is the line currents entering the shunt components. These line currents will be used in the forward-backward sweep method. The approaches to calculate these line currents depend on the components and on wye/delta connections.*

Wye-Connected Loads

Fig.1 shows the model of a wye-connected load.

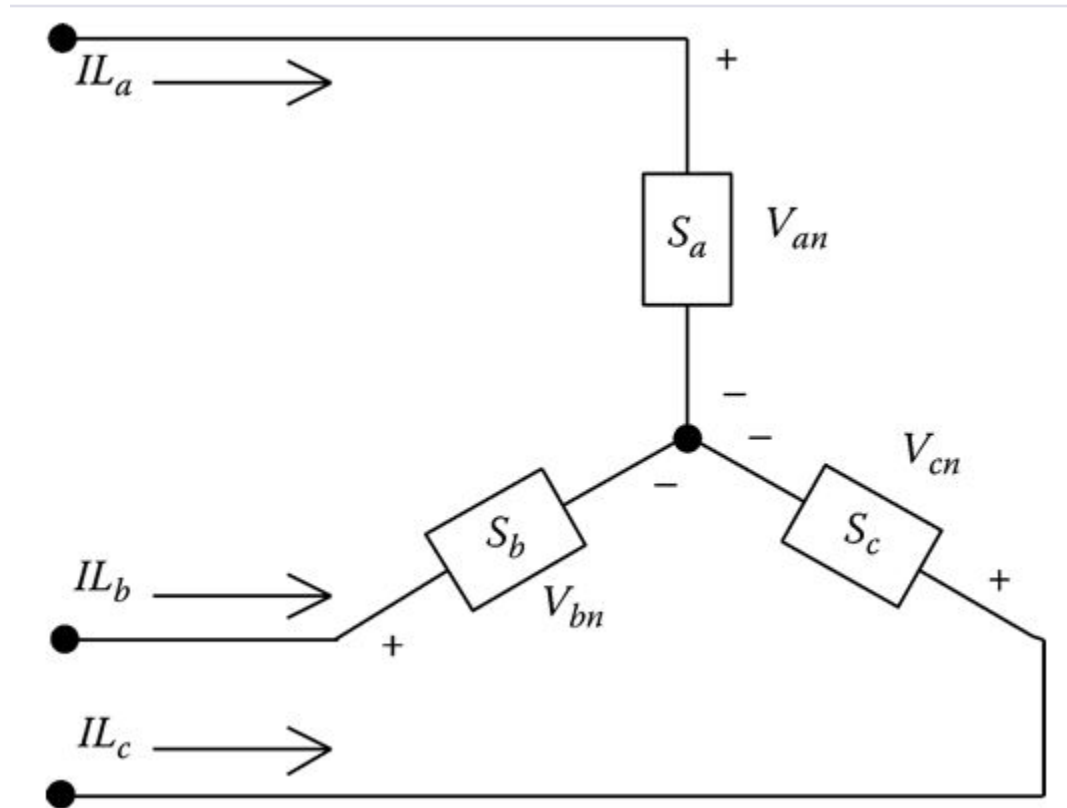


Fig.1 Wye-connected load

Wye-Connected Loads

The notation for the specified complex powers and voltages are as follows:

$$\text{Phase } a: \quad |S_a| \angle \theta_a = P_a + jQ_a \text{ and } |V_{an}| \angle \delta_a \quad (1)$$

$$\text{Phase } b: \quad |S_b| \angle \theta_b = P_b + jQ_b \text{ and } |V_{bn}| \angle \delta_b \quad (2)$$

$$\text{Phase } c: \quad |S_c| \angle \theta_c = P_c + jQ_c \text{ and } |V_{cn}| \angle \delta_c \quad (3)$$

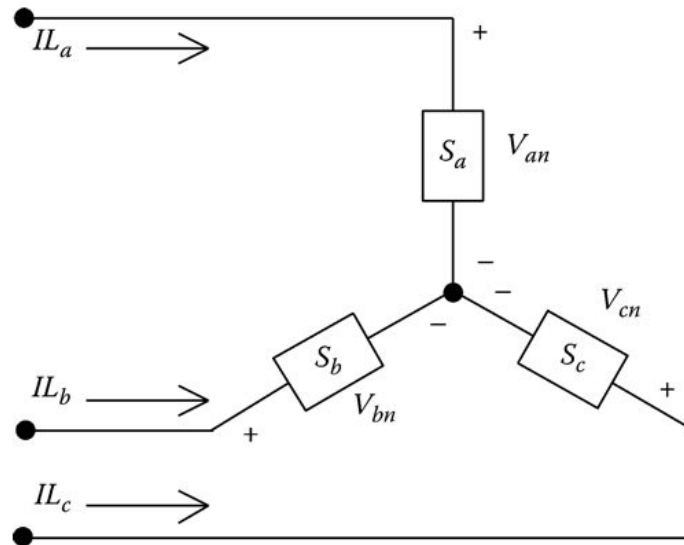


Fig.1 Wye-connected load

Constant Real and Reactive Power Loads

The line currents for constant real and reactive power loads (PQ loads) are given by

$$\begin{aligned} IL_a &= \left(\frac{S_a}{V_{an}} \right)^* = \frac{|S_a|}{|V_{an}|} \angle \delta_a - \theta_a = |IL_a| \angle \delta_a \\ IL_b &= \left(\frac{S_b}{V_{bn}} \right)^* = \frac{|S_b|}{|V_{bn}|} \angle \delta_b - \theta_b = |IL_b| \angle \delta_b \\ IL_c &= \left(\frac{S_c}{V_{cn}} \right)^* = \frac{|S_c|}{|V_{cn}|} \angle \delta_c - \theta_c = |IL_c| \angle \delta_c \end{aligned} \quad (4)$$

In this model, the line-to-neutral voltages will change during each iteration until convergence is achieved.

Constant Impedance Loads

The “constant load impedance” is first determined from the specified complex power and **assumed (e.g., nominal voltage)** line-to-neutral voltages:

$$\begin{aligned}Z_a &= \frac{|V_{an}|^2}{S_a^*} = \frac{|V_{an}|^2}{|S_a|} \angle \theta_a = |Z_a| \angle \theta_a \\Z_b &= \frac{|V_{bn}|^2}{S_b^*} = \frac{|V_{bn}|^2}{|S_b|} \angle \theta_b = |Z_b| \angle \theta_b \\Z_c &= \frac{|V_{cn}|^2}{S_c^*} = \frac{|V_{cn}|^2}{|S_c|} \angle \theta_c = |Z_c| \angle \theta_c\end{aligned}\tag{5}$$

The load currents as a function of the “constant load impedances” are given by

$$\begin{aligned}IL_a &= \frac{V_{an}}{Z_a} = \frac{|V_{an}|}{|Z_a|} \angle \delta_a - \theta_a = |IL_a| \angle \alpha_a \\IL_b &= \frac{V_{bn}}{Z_b} = \frac{|V_{bn}|}{|Z_b|} \angle \delta_b - \theta_b = |IL_b| \angle \alpha_b \\IL_c &= \frac{V_{cn}}{Z_c} = \frac{|V_{cn}|}{|Z_c|} \angle \delta_c - \theta_c = |IL_c| \angle \alpha_c\end{aligned}\tag{6}$$

In this model, the line-to-neutral voltages will change during each iteration, but the impedance computed in Equation (5) will remain constant.

Constant Current Loads

$$\begin{aligned} IL_a &= \left(\frac{S_a}{V_{an}} \right)^* = \frac{|S_a|}{|V_{an}|} \angle \delta_a - \theta_a = |IL_a| \angle \delta_a \\ IL_b &= \left(\frac{S_b}{V_{bn}} \right)^* = \frac{|S_b|}{|V_{bn}|} \angle \delta_b - \theta_b = |IL_b| \angle \delta_b \\ IL_c &= \left(\frac{S_c}{V_{cn}} \right)^* = \frac{|S_c|}{|V_{cn}|} \angle \delta_c - \theta_c = |IL_c| \angle \delta_c \end{aligned} \quad (4)$$

In this model, the **magnitudes** of the currents are computed according to Equations (4) and then **held constant** while the angle of the voltage (δ) changes resulting in a changing angle on the current so that the **power factor of the load remains constant**:

$$\begin{aligned} IL_a &= |IL_a| \angle \delta_a - \theta_a \\ IL_b &= |IL_b| \angle \delta_b - \theta_b \\ IL_c &= |IL_c| \angle \delta_c - \theta_c \end{aligned} \quad (7)$$

where

δ_{abc} represents the line-to-neutral voltage angles

θ_{abc} represents the power factor angles

Combination Loads

Combination loads can be modeled by assigning a percentage of the total load to each of the three aforementioned load models. The total line current entering the load is the sum of the three components.

Example 1

The complex powers of a wye-connected load are

$$[S_{abc}] = \begin{bmatrix} 2236.1 \angle 26.6 \\ 2506.0 \angle 28.6 \\ 2101.4 \angle 25.3 \end{bmatrix} \text{ kVA}$$

The load is specified to be 50% constant complex power, 20% constant impedance, and 30% constant current. The nominal line-to-line voltage of the feeder is 12.47 kV.

A. Assume the nominal voltage and compute the component of load current attributed to each component of the load and the total load current. The assumed line-to-neutral voltages at the start of the iterative routine are

$$[VLN_{abc}] = \begin{bmatrix} 7200 \angle 0 \\ 7200 \angle -120 \\ 7200 \angle 120 \end{bmatrix} \text{ V}$$

The component of currents due to the constant complex power is

$$I_{pq_i} = \left(\frac{S_i \cdot 1000}{VLN_i} \right)^* \cdot 0.5 = \begin{bmatrix} 155.3 \angle -26.6 \\ 174.0 \angle -148.6 \\ 146.0 \angle 94.7 \end{bmatrix} \text{ A}$$

Example 1

The constant impedances for that part of the load are computed as

$$Z_i = \frac{VLN_i^2}{S_i^* \cdot 1000} \cdot 0.5 = \begin{bmatrix} 20.7 + j10.4 \\ 18.2 + j9.9 \\ 22.3 + j10.6 \end{bmatrix} \Omega$$

For the first iteration, the currents due to the constant impedance portion of the load are

$$I_{Z_i} = \left(\frac{VLN_i}{Z_i} \right) \cdot 0.2 = \begin{bmatrix} 62.1 \angle -26.6 \\ 69.6 \angle -148.6 \\ 58.4 \angle 94.7 \end{bmatrix} A$$

The magnitudes of the constant current portion of the load are

$$IM_i = \left| \left(\frac{S_i \cdot 1000}{VLN_i} \right)^* \right| \cdot 0.3 = \begin{bmatrix} 93.2 \\ 104.4 \\ 87.6 \end{bmatrix} A$$

Example 1

The contribution of the load currents due to the constant current portion of the load is

$$II_i = IM_i \angle \delta_i - \theta_i = \begin{bmatrix} 93.2 \angle -26.6 \\ 104.4 \angle -148.6 \\ 87.6 \angle 94.7 \end{bmatrix} A$$

The total load current is the sum of the three components:

$$[I_{abc}] = [I_{pq}] + [I_z] + [I_I] = \begin{bmatrix} 310.6 \angle -26.6 \\ 348.1 \angle -148.6 \\ 292.0 \angle 94.7 \end{bmatrix} A$$

B. Determine the currents at the start of the second iteration. The voltages at the load after the first iteration are

$$[VLN] = \begin{bmatrix} 6850.0 \angle -1.9 \\ 6972.7 \angle -122.1 \\ 6886.1 \angle 117.5 \end{bmatrix} V$$

The steps are repeated with the exceptions that the impedances of the constant impedance portion of the load will not be changed and the magnitude of the currents for the constant current portion of the load change will not change.

Example 1

The constant complex power portion of the load currents is

$$I_{pq_i} = \left(\frac{S_i \cdot 1000}{VLN_i} \right)^* \cdot 0.5 = \begin{bmatrix} 163.2 \angle -28.5 \\ 179.7 \angle -150.7 \\ 152.7 \angle 92.1 \end{bmatrix} A$$

The currents due to the constant impedance portion of the load are

$$I_{z_i} = \left(\frac{VLN_i}{Z_i} \right) \cdot 0.2 = \begin{bmatrix} 59.1 \angle -28.5 \\ 67.4 \angle -150.7 \\ 55.9 \angle 92.1 \end{bmatrix} A$$

The currents due to the constant current portion of the load are

$$I_{I_i} = IM_i \angle \delta_i - \theta_i = \begin{bmatrix} 93.2 \angle -28.5 \\ 104.4 \angle -150.7 \\ 87.6 \angle 92.1 \end{bmatrix} A$$

The total load currents at the start of the second iteration will be

$$[I_{abc}] = [I_{pq}] + [I_z] + [I_I] = \begin{bmatrix} 315.5 \angle -28.5 \\ 351.5 \angle -150.7 \\ 296.2 \angle 92.1 \end{bmatrix} A$$

Example 1

Observe how these currents have changed from the original currents. The currents for the constant complex power loads have increased because the voltages are reduced from the original assumption. The currents for the constant impedance portion of the load have decreased because the impedance stayed constant but the voltages are reduced. Finally, the constant current portion of the load has indeed remained constant. Again, all three components of the load have the same phase angles since the power factor of the load has not changed.

Delta-Connected Loads

The model for a delta-connected load is shown in Fig.2.

The notations for the specified complex powers and voltages in Fig.2 are as follows:

Phase *ab*: $|S_{ab}| \angle \theta_{ab} = P_{ab} + jQ_{ab}$ and $|V_{ab}| \angle \delta_{ab}$ (8)

Phase *bc*: $|S_{bc}| \angle \theta_{bc} = P_{bc} + jQ_{bc}$ and $|V_{bc}| \angle \delta_{bc}$ (9)

Phase *ca*: $|S_{ca}| \angle \theta_{ca} = P_{ca} + jQ_{ca}$ and $|V_{ca}| \angle \delta_{ca}$ (10)

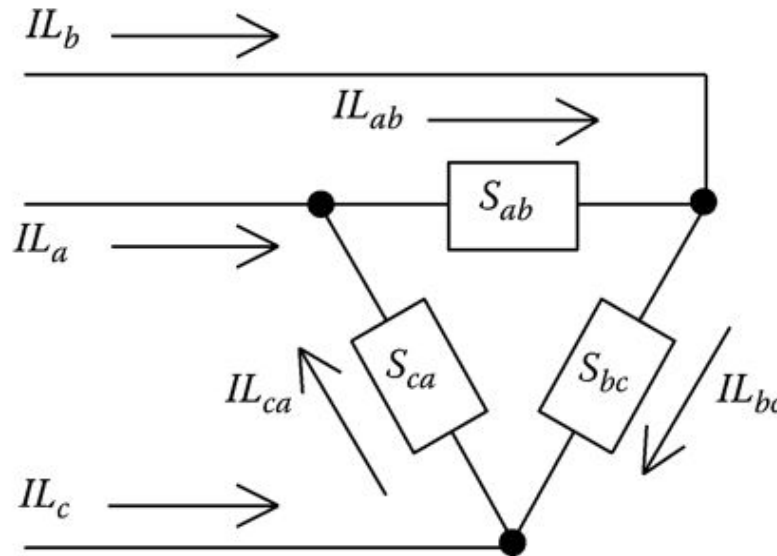


Fig.2 Delta-connected load

Constant Real and Reactive Power Loads

The currents in the delta-connected loads are

$$\begin{aligned} I_{L_{ab}} &= \left(\frac{S_{ab}}{V_{ab}} \right)^* = \frac{|S_{ab}|}{|V_{ab}|} \angle \delta_{ab} - \theta_{ab} = |I_{L_{ab}}| \angle \alpha_{ab} \\ I_{L_{bc}} &= \left(\frac{S_{bc}}{V_{bc}} \right)^* = \frac{|S_{bc}|}{|V_{bc}|} \angle \delta_{bc} - \theta_{bc} = |I_{L_{bc}}| \angle \alpha_{bc} \\ I_{L_{ca}} &= \left(\frac{S_{ca}}{V_{ca}} \right)^* = \frac{|S_{ca}|}{|V_{ca}|} \angle \delta_{ca} - \theta_{ca} = |I_{L_{ca}}| \angle \alpha_{ca} \end{aligned} \quad (11)$$

In this model, the line-to-line voltages will change during each iteration resulting in new current magnitudes and angles at the start of each iteration.

Constant Impedance Loads

The “constant load impedance” is first determined from the specified complex power and line-to-line voltages:

$$\begin{aligned}Z_{ab} &= \frac{|V_{ab}|^2}{S_{ab}^*} = \frac{|V_{ab}|^2}{|S_{ab}|} \angle \theta_{ab} = |Z_{ab}| \angle \theta_{ab} \\Z_{bc} &= \frac{|V_{bc}|^2}{S_{bc}^*} = \frac{|V_{bc}|^2}{|S_{bc}|} \angle \theta_{bc} = |Z_{bc}| \angle \theta_{bc} \\Z_{ca} &= \frac{|V_{ca}|^2}{S_{ca}^*} = \frac{|V_{ca}|^2}{|S_{ca}|} \angle \theta_{ca} = |Z_{ca}| \angle \theta_{ca}\end{aligned}\tag{12}$$

The delta load currents as a function of the “constant load impedances” are

$$\begin{aligned}IL_{ab} &= \frac{V_{ab}}{Z_{ab}} = \frac{|V_{ab}|}{|Z_{ab}|} \angle \delta_{ab} - \theta_{ab} = |IL_{ab}| \angle \alpha_{ab} \\IL_{bc} &= \frac{V_{bc}}{Z_{bc}} = \frac{|V_{bc}|}{|Z_{bc}|} \angle \delta_{bc} - \theta_{bc} = |IL_{bc}| \angle \alpha_{bc} \\IL_{ca} &= \frac{V_{ca}}{Z_{ca}} = \frac{|V_{ca}|}{|Z_{ca}|} \angle \delta_{ca} - \theta_{ca} = |IL_{ca}| \angle \alpha_{ca}\end{aligned}\tag{13}$$

In this model, the line-to-line voltages will change during each iteration until convergence is achieved.

Constant Current Loads

$$\begin{aligned} I_{L_{ab}} &= \left(\frac{S_{ab}}{V_{ab}} \right)^* = \frac{|S_{ab}|}{|V_{ab}|} \angle \delta_{ab} - \theta_{ab} = |I_{L_{ab}}| \angle \alpha_{ab} \\ I_{L_{bc}} &= \left(\frac{S_{bc}}{V_{bc}} \right)^* = \frac{|S_{bc}|}{|V_{bc}|} \angle \delta_{bc} - \theta_{bc} = |I_{L_{bc}}| \angle \alpha_{bc} \\ I_{L_{ca}} &= \left(\frac{S_{ca}}{V_{ca}} \right)^* = \frac{|S_{ca}|}{|V_{ca}|} \angle \delta_{ca} - \theta_{ca} = |I_{L_{ca}}| \angle \alpha_{ca} \end{aligned} \quad (11)$$

In this model, the magnitudes of the currents are computed according to Equations (11) and then held constant while the angle of the voltage (δ) changes during each iteration. This keeps the power factor of the load constant:

$$\begin{aligned} I_{L_{ab}} &= |I_{L_{ab}}| \angle \delta_{ab} - \theta_{ab} \\ I_{L_{bc}} &= |I_{L_{bc}}| \angle \delta_{bc} - \theta_{bc} \\ I_{L_{ca}} &= |I_{L_{ca}}| \angle \delta_{ca} - \theta_{ca} \end{aligned} \quad (14)$$

Combination Loads

Combination loads can be modeled by assigning a percentage of the total load to each of the three aforementioned load models. The total delta current for each load is the sum of the three components.

Line Currents Serving a Delta-Connected Loads

The line currents entering the delta-connected load are determined by applying Kirchhoff's current law (KCL) at each of the nodes of the delta. In matrix form, the equations are

$$\begin{bmatrix} IL_a \\ IL_b \\ IL_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} IL_{ab} \\ IL_{bc} \\ IL_{ca} \end{bmatrix} \quad (15)$$

Two-Phase and Single-Phase Loads

In both the wye- and delta-connected loads, single-phase and two-phase loads are modeled by setting the currents of the missing phases to zero. The currents in the phases present are computed using the same appropriate equations for constant complex power, constant impedance, and constant current.

Shunt Capacitors

Shunt capacitor banks are commonly used in distribution systems to help in voltage regulation and to provide reactive power support. The capacitor banks are modeled as constant susceptances connected in either wye or delta. Similar to the load model, all capacitor banks are modeled as three-phase banks with the currents of the missing phases set to zero for single-phase and two-phase banks.

Wye-Connected Capacitor Bank

The model of a three-phase wye-connected shunt capacitor bank is shown in Fig.3. The individual phase capacitor units are specified in kvar and kV. The constant susceptance for each unit can be computed in Siemens. The susceptance of a capacitor unit is computed by

$$B_c = \frac{kvar}{kV_{LN}^2 \cdot 1000} S \quad (16)$$

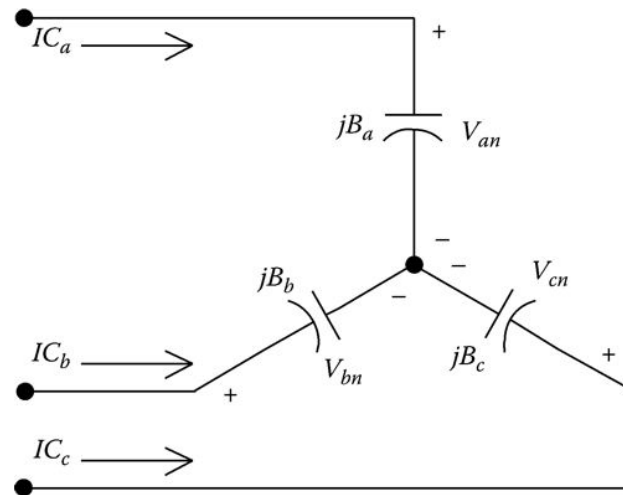


Fig.3 Wye-connected capacitor bank

Wye-Connected Capacitor Bank

With the susceptance computed, the line currents serving the capacitor bank are given by

$$\begin{aligned} IC_a &= jB_a \cdot V_{an} \\ IC_b &= jB_b \cdot V_{bn} \\ IC_c &= jB_c \cdot V_{cn} \end{aligned} \quad (17)$$

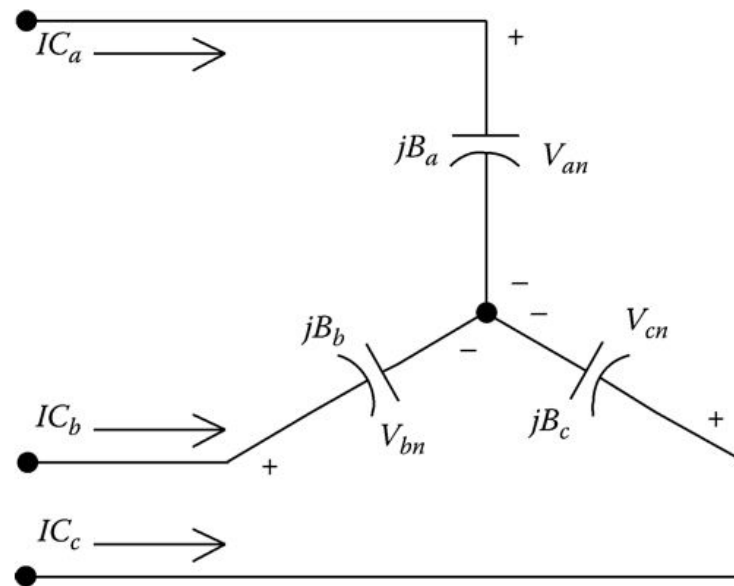


Fig.3 Wye-connected capacitor bank

Delta-Connected Capacitor Bank

The model for a delta-connected shunt capacitor bank is shown in Fig.4.

The individual phase capacitor units are specified in kvar and kV. For the delta-connected capacitors, the kV must be the line-to-line voltage. The constant susceptance for each unit can be computed in Siemens. The susceptance of a capacitor unit is computed by

$$B_c = \frac{kvar}{kV_{LL}^2 \cdot 1000} S \quad (18)$$

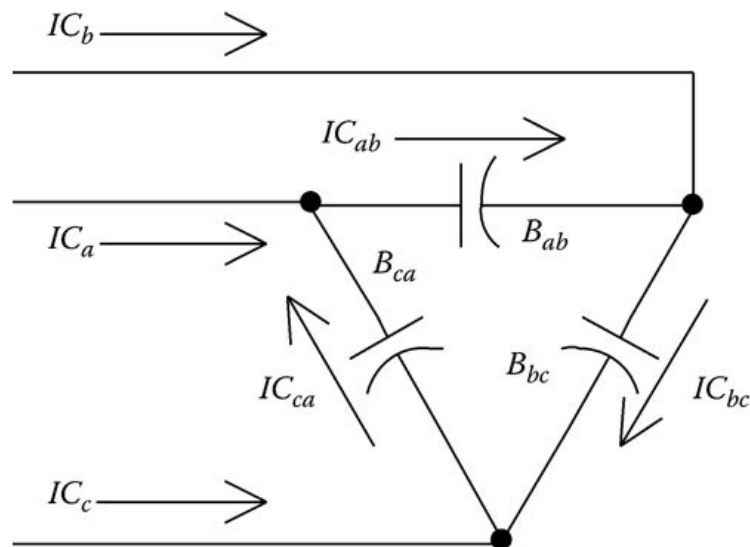


Fig.4 Delta-connected capacitor bank

Delta-Connected Capacitor Bank

With the susceptance computed, the delta currents serving the capacitor bank are given by

$$\begin{aligned}IC_{ab} &= jB_{ab} \cdot V_{ab} \\IC_{bc} &= jB_{bc} \cdot V_{bc} \\IC_{ca} &= jB_{ca} \cdot V_{ca}\end{aligned}\tag{19}$$

The line currents flowing into the delta-connected capacitors are computed by applying KCL at each node. In matrix form, the equations are

$$\begin{bmatrix}IC_a \\IC_b \\IC_c\end{bmatrix} = \begin{bmatrix}1 & 0 & -1 \\-1 & 1 & 0 \\0 & -1 & 1\end{bmatrix} \cdot \begin{bmatrix}IC_{ab} \\IC_{bc} \\IC_{ca}\end{bmatrix}\tag{20}$$

Three-Phase Induction Machine

The analysis of an induction machine (motor or generator) when operating under unbalanced voltage conditions has traditionally been performed using the method of symmetrical components. Using this approach, the positive and negative sequence equivalent circuits of the machine are developed and then, given the sequence line-to-neutral voltages, the sequence currents are computed. The zero sequence network is not required since the machines are typically connected delta or ungrounded wye, which means that there will not be any zero sequence currents or voltages. The phase currents are determined by performing the transformation back to the phase line currents. The internal operating conditions are determined by the complete analysis of the sequence networks.

A method whereby all of the analysis can be performed in the phase frame will be developed. The analysis will be broken into two parts. The first part will be to determine the terminal voltages and currents of the motor and the second part will be to use these values to compute the stator and rotor losses and the converted shaft power.

Induction Machine Model

The sequence line-to-neutral equivalent circuit of a three-phase induction machine is shown in Fig.5.

The circuit in Fig.5 applies to both the positive and negative sequence networks. The only difference between the two is the value of the “load resistance” RL as defined in the following:

$$RL_i = \frac{1 - s_i}{s_i} \cdot Rr_i \quad (21)$$

where

Positive sequence slip:

$$s_i = \frac{n_s - n_r}{n_s} \quad (22)$$

where

- n_s is the synchronous speed
- n_r is the rotor speed

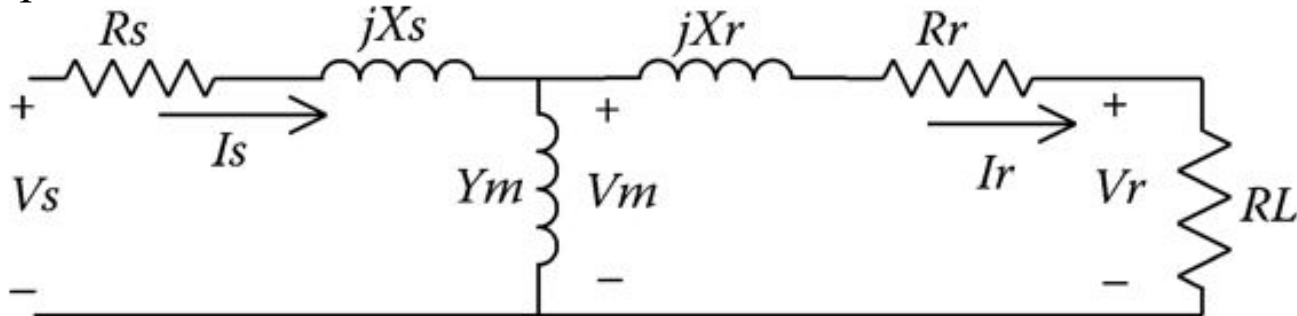


Fig.5 Sequence equivalent circuit

Induction Machine Model

Negative sequence slip:

$$s_2 = 2 - s_1 \quad (23)$$

Note that the negative sequence load resistance RL_2 will be a negative value that will lead to a negative shaft power in the negative sequence.

If the value of positive sequence slip (s_1) is known, the input sequence impedances for the positive and negative sequence networks can be determined as

$$ZM_i = Rs_i + jXs_i + \frac{(jXm_i)(Rr_i + RL_i + jXr_i)}{Rr_i + RL_i + j(Xm_i + Xr_i)} \quad (24)$$

where

- $i = 1$ for positive sequence
- $i = 2$ for negative sequence

Induction Machine Model

Once the input sequence impedances have been determined, the analysis of an induction machine operating with unbalanced voltages requires the following steps:

Step 1: Transform the known line-to-line voltages to sequence line-to-line voltages:

$$\begin{bmatrix} Vab_0 \\ Vab_1 \\ Vab_2 \end{bmatrix} = \frac{1}{3} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \cdot \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} \quad (25)$$

In Equation (25), $Vab_0 = 0$ because of Kirchhoff's voltage law (KVL).

Equation (25) can be written as

$$[VLL_{012}] = [A]^{-1} \cdot [VLL_{abc}] \quad (26)$$

Step 2: Compute the sequence line-to-neutral voltages from the line-to-line voltages:

$$Van_0 = Vab_0 = 0 \quad (27)$$

Equation (27) will not be true for a general case. However, for the case of the machine being connected either in delta or ungrounded wye, the zero sequence line-to-neutral voltage can be assumed to be zero:

$$Van_1 = t^* \cdot Vab_1 \quad (28)$$

$$Van_2 = t^* \cdot Vab_2 \quad (29)$$

where

$$t = \frac{1}{\sqrt{3}} \cdot \angle 30 \quad (30)$$

Induction Machine Model

$$Van_0 = Vab_0 = 0 \quad (27)$$

$$Van_1 = t^*Vab_1 \quad (28)$$

$$Van_2 = tVab_2 \quad (29)$$

Equations (27) through (29) can be put into matrix form:

$$\begin{bmatrix} Van_0 \\ Van_1 \\ Van_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & t^* & 0 \\ 0 & 0 & t \end{bmatrix} \cdot \begin{bmatrix} Vab_0 \\ Vab_1 \\ Vab_2 \end{bmatrix} \quad (31)$$

Equations (31) can be written as

$$[VLN_{012}] = [T] \cdot [VLL_{012}] \quad (32)$$

Step 3: Compute the sequence line currents flowing into the machine:

$$Ia_0 = 0 \quad (33)$$

$$Ia_1 = \frac{Van_1}{ZM_1} \quad (34)$$

$$Ia_2 = \frac{Van_2}{ZM_2} \quad (35)$$

Induction Machine Model

Step 4: Transform the sequence currents to phase currents:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \cdot \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} \quad (36)$$

Equation (36) can be written as

$$[I_{abc}] = [A] \cdot [I_{012}] \quad (37)$$

The four steps outlined earlier can be performed without actually computing the sequence voltages and currents. The procedure basically reverses the steps.

Define

$$YM_i = \frac{1}{ZM_i} \quad (38)$$

The sequence currents are

$$I_0 = 0 \quad (40)$$

$$I_1 = YM_1 \cdot Van_1 = YM_1 \cdot t^* \cdot Vab_1 \quad (41)$$

$$I_2 = YM_2 \cdot Van_2 = YM_2 \cdot t^* \cdot Vab_2$$

Since I_0 and Vab_0 are both zero, the following relationship is true:

$$I_0 = Vab_0 \quad (42)$$

Induction Machine Model

$$I_0 = 0 \quad (39)$$

$$I_1 = YM_1 \cdot Van_1 = YM_1 \cdot t^* \cdot Vab_1 \quad (40)$$

$$I_0 = Vab_0 \quad (42)$$

Equations (39), (40), and (42) can be put into matrix form:

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & t^* \cdot YM_1 & 0 \\ 0 & 0 & t \cdot YM_1 \end{bmatrix} \cdot \begin{bmatrix} Vab_0 \\ Vab_1 \\ Vab_2 \end{bmatrix} \quad (43)$$

Equation (43) can be written in shortened form as

$$[I_{012}] = [YM_{012}] \cdot [VLL_{012}] \quad (44)$$

From symmetrical component theory,

$$[VLL_{012}] = [A]^{-1} \cdot [VLL_{abc}] \quad (45)$$

$$[I_{abc}] = [A] \cdot [I_{012}] \quad (46)$$

Substitute Equation (45) into Equation (44) and substitute the resultant equation into Equation (46) to get

$$[I_{abc}] = [A] \cdot [YM_{012}] \cdot [A]^{-1} \cdot [VLL_{abc}] \quad (47)$$

Induction Machine Model

Define $[YM_{abc}] = [A] \cdot [YM_{012}] \cdot [A]^{-1}$ (48)

Therefore $[I_{abc}] = [YM_{abc}] \cdot [VLL_{abc}]$ (49)

The induction machine “phase frame admittance matrix” $[YM_{abc}]$ is defined in Equation (48). Equation (49) is used to compute the input phase currents of the machine from a knowledge of the phase line-to-line terminal voltages. This is the desired result. Recall that $[YM_{abc}]$ is a function of the slip of the machine so that a new matrix must be computed every time the slip changes.

Equation (49) can be used to solve for the line-to-line voltages as a function of the line currents by

$$[VLL_{abc}] = [ZM_{abc}] \cdot [I_{abc}] \quad (50)$$

where $[ZM_{abc}] = [YM_{abc}]^{-1}$ (51)

As was done in Chapter 8, it is possible to replace the line-to-line voltages in Equation (50) with the “equivalent” line-to-neutral voltages:

$$[VLN_{abc}] = [W] \cdot [VLL_{abc}] \quad (52)$$

Induction Machine Model

Define

$$[W] = [A] \cdot [T] \cdot [A]^{-1} \quad (53)$$

The matrix $[W]$ is a very useful matrix that allows the determination of the “equivalent” line-to-neutral voltages from a knowledge of the line-to-line voltages. Equation (50) can be substituted into Equation (52) to define the “line-to-neutral” equation:

$$[VLN_{abc}] = [W] \cdot [ZM_{abc}] \cdot [I_{abc}] \quad (54)$$

$$[VLN_{abc}] = [ZLN_{abc}] \cdot [I_{abc}]$$

where

$$[ZLN_{abc}] = [W] \cdot [ZM_{abc}] \quad (55)$$

The inverse of Equation (54) can be taken to determine the line currents as a function of the line-to-neutral voltages:

$$[I_{abc}] = [YLN_{abc}] \cdot [VLN_{abc}] \quad (56)$$

where

$$[YLN_{abc}] = [ZLN_{abc}]^{-1} \quad (57)$$

Induction Machine Model

$$[VLN_{abc}] = [W] \cdot [VLL_{abc}] \quad (52)$$

$$[I_{abc}] = [YLN_{abc}] \cdot [VLN_{abc}] \quad (56)$$

Care must be taken in applying Equation (56) to insure that the voltages used are the line-to-neutral and not the line-to-ground voltages. If only the line-to-ground voltages are known, they must first be converted to the line-to-line values and then use Equation (52) to compute the line-to-neutral voltages.

Once the machine terminal currents and line-to-neutral voltages are known, the input phase complex powers and total three-phase input complex power can be computed:

$$S_a = V_{an} \cdot (I_a)^* \quad (58)$$

$$S_b = V_{bn} \cdot (I_b)^* \quad (59)$$

$$S_c = V_{cn} \cdot (I_c)^* \quad (60)$$

$$S_{Total} = S_a + S_b + S_c \quad (61)$$

Many times the only voltages known will be the magnitudes of the three line-to-line voltages at the machine terminals. When this is the case, the Law of Cosines must be used to compute the angles associated with the measured magnitudes.

Equivalent T circuit

Once the terminal line-to-neutral voltages and currents are known, it is desired to analyze what is happening inside the machine. In particular, the stator and rotor losses are needed in addition to the “converted” shaft power. A method of performing the internal analysis can be developed in the phase frame by starting with the sequence networks. Fig.5 can be modified by removing RL , which represents the “load resistance” in the positive and negative sequence networks. The resulting networks will be modeled using A , B , C , and D parameters. The equivalent T circuit (RL removed) is shown in Fig.6. This circuit can represent both the positive and negative sequence networks. The only difference (if any) will be between the numerical values of the sequence stator and rotor impedances.

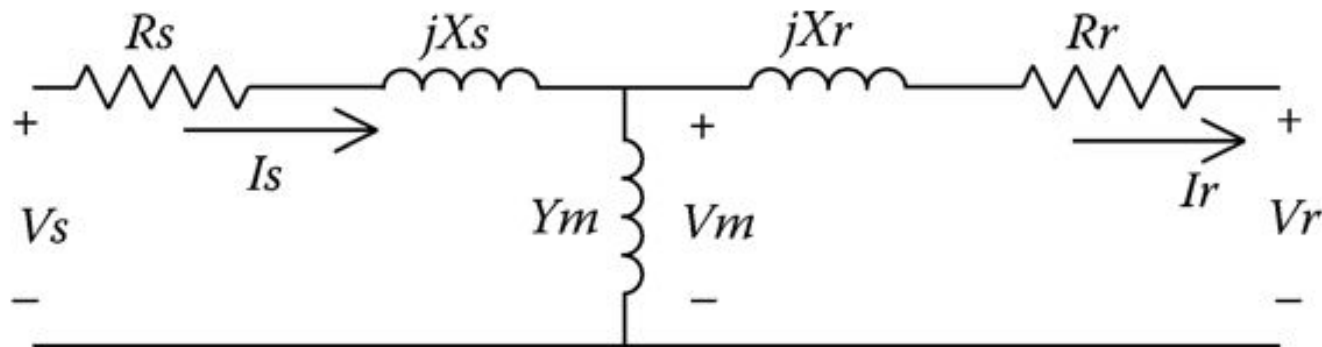


Fig.6 Equivalent T circuit

Equivalent T circuit

In Fig.6,

$$Y_m = \frac{1}{jX_m} \quad (62)$$

Define the sequence stator and rotor impedances:

$$Z_{s_i} = R_{s_i} + jX_{s_i} \quad (63)$$

$$Z_{r_i} = R_{r_i} + jX_{r_i} \quad (64)$$

The positive and negative sequence A , B , C , and D parameters of the unsymmetrical T circuit of Fig.6 are given by

$$A_{m_i} = 1 + Y_{m_i} \cdot Z_{s_i} \quad (65)$$

$$B_{m_i} = Z_{s_i} + Z_{r_i} + Y_{m_i} \cdot Z_{s_i} \cdot Z_{r_i} \quad (66)$$

$$C_{m_i} = Y_{m_i} \quad (67)$$

$$D_{m_i} = 1 + Y_{m_i} \cdot Z_{r_i} \quad (68)$$

where

- $i = 1$ for positive sequence
- $i = 2$ for negative sequence

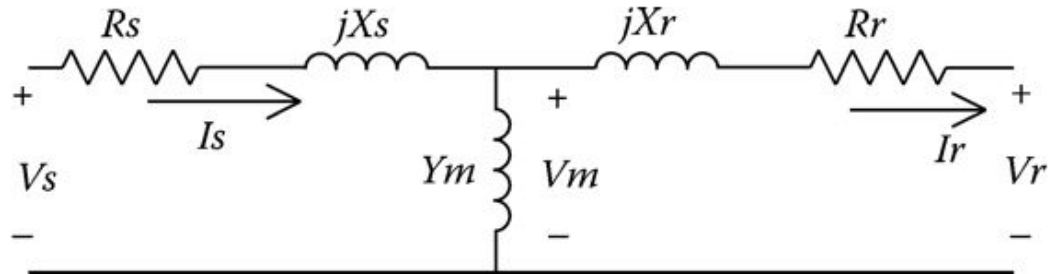


Fig.6 Equivalent T circuit

Equivalent T circuit

Note
$$Am_i \cdot Dm_i - Bm_i \cdot Cm_i = 1 \quad (69)$$

The terminal sequence line-to-neutral voltages and currents as functions of the rotor “load voltages” (Vr) and the rotor currents are given by

$$\begin{bmatrix} Vs_i \\ Is_i \end{bmatrix} = \begin{bmatrix} Am_i & Bm_i \\ Cm_i & Dm_i \end{bmatrix} \cdot \begin{bmatrix} Vr_i \\ Ir_i \end{bmatrix} \quad (70)$$

Because of Equation (69), the inverse of Equation (70) is

$$\begin{bmatrix} Vr_i \\ Ir_i \end{bmatrix} = \begin{bmatrix} Dm_i & -Bm_i \\ -Cm_i & Am_i \end{bmatrix} \cdot \begin{bmatrix} Vs_i \\ Is_i \end{bmatrix} \quad (71)$$

Equation (71) can be expanded to show the individual sequence voltages and currents:

$$\begin{bmatrix} Vr_0 \\ Vr_1 \\ Vr_2 \\ Ir_0 \\ Ir_1 \\ Ir_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Dm_1 & 0 & 0 & -Bm_1 & 0 \\ 0 & 0 & Dm_1 & 0 & 0 & -Bm_2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -Cm_1 & 0 & 0 & Am_1 & 0 \\ 0 & 0 & -Cm_2 & 0 & 0 & Am_2 \end{bmatrix} \cdot \begin{bmatrix} Vs_0 \\ Vs_1 \\ Vs_2 \\ Is_0 \\ Is_1 \\ Is_2 \end{bmatrix} \quad (72)$$

Equivalent T circuit

$$\begin{bmatrix} Vr_0 \\ Vr_1 \\ Vr_2 \\ Ir_0 \\ Ir_1 \\ Ir_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Dm_1 & 0 & 0 & -Bm_1 & 0 \\ 0 & 0 & Dm_1 & 0 & 0 & -Bm_2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -Cm_1 & 0 & 0 & Am_1 & 0 \\ 0 & 0 & -Cm_2 & 0 & 0 & Am_2 \end{bmatrix} \cdot \begin{bmatrix} Vs_0 \\ Vs_1 \\ Vs_2 \\ Is_0 \\ Is_1 \\ Is_2 \end{bmatrix} \quad (72)$$

Equation (72) can be partitioned between the third and fourth rows and columns. In reduced form by incorporating the partitioning, Equation (72) becomes

$$\begin{bmatrix} [Vr_{012}] \\ [Ir_{012}] \end{bmatrix} = \begin{bmatrix} [Dm_{012}] & [Bm_{012}] \\ [Cm_{012}] & [Am_{012}] \end{bmatrix} \cdot \begin{bmatrix} [Vs_{012}] \\ [Is_{012}] \end{bmatrix} \quad (73)$$

Expanding Equation (73),

$$[Vr_{012}] = [Dm_{012}] \cdot [Vs_{012}] + [Bm_{012}] \cdot [Is_{012}] \quad (74)$$

$$[Ir_{012}] = [Cm_{012}] \cdot [Vs_{012}] + [Am_{012}] \cdot [Is_{012}] \quad (75)$$

Equations (74) and (75) can be transformed into the phase domain:

$$[Vr_{abc}] = [A] \cdot [Vr_{012}] = [A] \cdot [Dm_{012}] \cdot [A]^{-1} \cdot [Vs_{abc}] + [A] \cdot [Bm_{012}] \cdot [A]^{-1} \cdot [Is_{abc}] \quad (76)$$

$$[Ir_{abc}] = [A] \cdot [Ir_{012}] = [A] \cdot [Cm_{012}] \cdot [A]^{-1} \cdot [Vs_{abc}] + [A] \cdot [Am_{012}] \cdot [A]^{-1} \cdot [Is_{abc}] \quad (77)_{39}$$

Equivalent T circuit

$$[Vr_{abc}] = [A] \cdot [Vr_{012}] = [A] \cdot [Dm_{012}] \cdot [A]^{-1} \cdot [Vs_{abc}] + [A] \cdot [Bm_{012}] \cdot [A]^{-1} \cdot [Is_{abc}] \quad (76)$$

$$[Ir_{abc}] = [A] \cdot [Ir_{012}] = [A] \cdot [Cm_{012}] \cdot [A]^{-1} \cdot [Vs_{abc}] + [A] \cdot [Am_{012}] \cdot [A]^{-1} \cdot [Is_{abc}] \quad (77)$$

Therefore

$$[Vr_{abc}] = [Dm_{abc}] \cdot [Vs_{abc}] + [Bm_{abc}] \cdot [Is_{abc}] \quad (78)$$

$$[Ir_{abc}] = [Cm_{abc}] \cdot [Vs_{abc}] + [Am_{abc}] \cdot [Is_{abc}] \quad (79)$$

where

$$[Am_{abc}] = [A] \cdot [Am_{012}] \cdot [A]^{-1}$$
$$[Bm_{abc}] = [A] \cdot [Bm_{012}] \cdot [A]^{-1} \quad (80)$$

$$[Cm_{abc}] = [A] \cdot [Cm_{012}] \cdot [A]^{-1}$$

$$[Dm_{abc}] = [A] \cdot [Dm_{012}] \cdot [A]^{-1}$$

The power converted to the shaft is given by

$$P_{conv} = Vr_a \cdot (Ir_a)^* + Vr_b \cdot (Ir_b)^* + Vr_c \cdot (Ir_c)^* \quad (81)$$

The useful shaft power can be determined from a knowledge of the rotational (FW) losses:

$$P_{shaft} = P_{conv} - P_{FW} \quad (82)$$

Equivalent T circuit

The rotor “copper” losses are

$$P_{rotor} = |I_{r_a}|^2 \cdot R_r + |I_{r_b}|^2 \cdot R_r + |I_{r_c}|^2 \cdot R_r \quad (83)$$

The stator “copper” losses are

$$P_{stator} = |I_{s_a}|^2 \cdot R_s + |I_{s_b}|^2 \cdot R_s + |I_{s_c}|^2 \cdot R_s \quad (84)$$

The total input power is

$$P_{in} = \text{Re}[V_{s_a} \cdot (I_{s_a})^* + V_{s_b} \cdot (I_{s_b})^* + V_{s_c} \cdot (I_{s_c})^*] \quad (85)$$

Example 2

To demonstrate the analysis of an induction motor in the phase frame, the following induction motor will be used:

- 25 hp, 240 V operating with $slip = 0.035$
- $P_{loss_{rotation}} = 0.75$ kW
- $Z_s = 0.0774 + j0.1843 \Omega$
- $Z_m = 0 + j4.8384 \Omega$
- $Z_r = 0.0908 + j0.1843 \Omega$

The “load” resistances are

$$RL_1 = \left(\frac{1 - 0.035}{0.035} \right) \cdot 0.098 = 2.5029 \Omega$$

$$RL_2 = \left(\frac{1 - (1.965)}{1.965} \right) \cdot 0.0908 = -0.0446 \Omega$$

The input sequence impedances are

$$ZM_1 = Z_s + \frac{Z_m \cdot (Z_r + RL_1)}{Z_m + Z_r + RL_1} = 1.9775 + j1.3431 \Omega$$

$$ZM_2 = Z_s + \frac{Z_m \cdot (Z_r + RL_2)}{Z_m + Z_r + RL_2} = 0.1203 + j0.3623 \Omega$$

Example 2

The positive and negative sequence input admittances are

$$YM_1 = \frac{1}{ZM_1} = 0.3461 - j0.2350 \text{ S}$$

$$YM_2 = \frac{1}{ZM_2} = 0.8255 - j2.4863 \text{ S}$$

The sequence admittance matrix is

$$[YM_{012}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.3461 - j0.2350 & 0 \\ 0 & 0 & 0.8255 - j2.4863 \end{bmatrix} \text{ S}$$

Applying Equation (48), the phase admittance matrix is

$$[YM_{abc}] = \begin{bmatrix} 0.7452 - j0.4074 & -0.0999 - j0.0923 & 0.3547 + j0.4997 \\ 0.3547 + j0.4997 & 0.7452 - j0.4074 & -0.0999 - j0.0923 \\ -0.0999 - j0.0923 & 0.3547 + j0.4997 & 0.7452 - j0.4074 \end{bmatrix} \text{ S}$$

The line-to-line terminal voltages are measured to be

$$V_{ab} = 235 \text{ V}, V_{bc} = 240 \text{ V}, V_{ca} = 245 \text{ V}$$

Example 2

Since the sum of the line-to-line voltages must equal zero, the law of cosines can be used to determine the angles on the voltages. Applying the law of cosines results in

$$[VLL_{abc}] = \begin{bmatrix} 235\angle 0 \\ 240\angle -117.9 \\ 245\angle 120.0 \end{bmatrix} V$$

The phase motor currents can now be computed:

$$[I_{S_{abc}}] = [Y_{M_{abc}}] \cdot [VLL_{abc}] = \begin{bmatrix} 53.15\angle -71.0 \\ 55.15\angle -175.1 \\ 66.6\angle 55.6 \end{bmatrix} A$$

It is obvious that the currents are very unbalanced. The measure of unbalance for the voltages and currents can be computed as [1]

$$V_{unbalance} = \left(\frac{\text{max_deviation}}{|V_{avg}|} \right) \cdot 100 = \left(\frac{5}{240} \right) \cdot 100 = 2.08\%$$
$$I_{unbalance} = \left(\frac{\text{max_deviation}}{|I_{avg}|} \right) \cdot 100 = \left(\frac{8.3232}{58.31} \right) \cdot 100 = 14.27\%$$

[1] American National Standard for Electric Power Systems and Equipment—Voltage Ratings (60 Hertz), ANSI C84.1-1995, National Electrical Manufacturers Association, Rosslyn, VA, 1996.

Example 2

This example demonstrates that the current unbalance is approximately seven times greater than the voltage unbalance. This ratio of current unbalance to voltage unbalance is typical. The actual operating characteristics including stator and rotor losses of the motor can be determined using the method developed in Ref. [2]

The equivalent line-to-neutral voltages at the motor are computed using the $[W]$ matrix:

$$[V_{S_{abc}}] = [W] \cdot [V_{LL_{abc}}]$$
$$\begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \frac{1}{3} \cdot \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 235.0 \angle 0 \\ 240 \angle -117.9 \\ 245 \angle 120.0 \end{bmatrix} = \begin{bmatrix} 138.6 \angle -30.7 \\ 135.7 \angle -148.6 \\ 141.4 \angle 91.4 \end{bmatrix}$$

The input complex power to the motor is

$$S_{in} = \sum_{k=1}^3 \frac{V_{S_{abc_k}} \cdot I_{abc_k}}{1000} = 19.95 + j13.62$$
$$|S_{in}| = 24.15 \quad PF = 0.83 \text{ lagging}$$

[2] Kersting, W.H. and Phillips, W.H., Phase frame analysis of the effects of voltage unbalance on induction machines, *IEEE Transactions on Industry Applications*, 33, 415–420, 1997.

Example 2

The rotor currents and voltages can be computed by first computing the equivalent A , B , C , and D matrices according to Equations (80). The first step is to compute the sequence A , B , C , and D parameters according to Equations (65) through (68):

$$Am_i = 1 + Ym_i \cdot Zs_i = 1.0381 - j0.0161$$

$$Bm_i = Zs_i + Zr_i + Ym_i \cdot Zs_i \cdot Zr_i = 0.1746 + j0.3742$$

$$Cm_i = Ym_i = -j0.2067$$

$$Dm_i = 1 + Ym_i \cdot Zr_i = 1.0381 - j0.0188$$

The sequence matrices using Equations (72) are

$$[Am_{012}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1.0381 - j0.0161 & 0 \\ 0 & 0 & 1.0381 - j0.0161 \end{bmatrix}$$

$$[Bm_{012}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -0.1746 - j0.3742 & 0 \\ 0 & 0 & -0.1746 - j0.3742 \end{bmatrix}$$

$$[Cm_{012}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & j0.2067 & 0 \\ 0 & 0 & j0.2067 \end{bmatrix} \quad [Dm_{012}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1.0381 - j0.0188 & 0 \\ 0 & 0 & 1.0381 - j0.0188 \end{bmatrix}$$

Example 2

Equation (80) gives us the final phase domain A , B , C , and D matrices:

$$[Am_{abc}] = [A] \cdot [Am_{012}] \cdot [A]^{-1} = \begin{bmatrix} 0.6921 - j0.0107 & -0.346 + j0.0053 & -0.346 + j0.0053 \\ -0.346 + j0.0053 & 0.6921 - j0.0107 & -0.346 + j0.0053 \\ -0.346 + j0.0053 & -0.346 + j0.0053 & 0.6921 - j0.0107 \end{bmatrix}$$

$$[Bm_{abc}] = [A] \cdot [Bm_{012}] \cdot [A]^{-1} = \begin{bmatrix} -0.1164 - j0.2494 & 0.0582 + j0.1247 & 0.0582 + j0.1247 \\ 0.0582 + j0.1247 & -0.1164 - j0.2494 & 0.0582 + j0.1247 \\ 0.0582 + j0.1247 & 0.0582 + j0.1247 & -0.1164 - j0.2494 \end{bmatrix}$$

$$[Cm_{abc}] = [A] \cdot [Cm_{012}] \cdot [A]^{-1} = \begin{bmatrix} j0.1378 & -j0.0689 & -j0.0689 \\ -j0.0689 & j0.1378 & -j0.0689 \\ -j0.0689 & -j0.0689 & j0.1378 \end{bmatrix}$$

$$[Dm_{abc}] = [A] \cdot [Dm_{012}] \cdot [A]^{-1} = \begin{bmatrix} 0.6921 - j0.0125 & -0.346 + j0.0063 & -0.346 + j0.0063 \\ -0.346 + j0.0063 & 0.6921 - j0.0125 & -0.346 + j0.0063 \\ -0.346 + j0.0063 & -0.346 + j0.0063 & 0.6921 - j0.0125 \end{bmatrix}$$

Example 2

With the matrices defined, the rotor voltages and currents can be computed:

$$[V_{r_{abc}}] = [Dm_{abc}] \cdot [Vs_{abc}] + [Bm_{abc}] \cdot [Is_{abc}] = \begin{bmatrix} 124.5\angle -36.1 \\ 124.1\angle -156.3 \\ 123.8\angle 83.9 \end{bmatrix}$$

$$[I_{r_{abc}}] = [Cm_{abc}] \cdot [Vs_{abc}] + [Am_{abc}] \cdot [Is_{abc}] = \begin{bmatrix} 42.2\angle -41.2 \\ 50.9\angle -146.6 \\ 56.8\angle 79.1 \end{bmatrix}$$

The converted electric power to shaft power is

$$P_{convert} = \sum_{k=1}^3 \frac{V_{s_{abc_k}} \cdot I_{abc_k}^*}{1000} = 18.5 \text{ kW}$$

The power converted in units of horsepower is

$$hp = \frac{P_{convert}}{0.746} = 24.8$$

Note how the shaft power in horsepower is approximately equal to the input kVA of the motor. This is typically the case so that a good assumption for a motor is that the rated output in horsepower will be equal to the input kVA.

Thank You!